Refinement Of Inventory Policy For Perishable Item Under Combined Effect Of Inflation, Limited Storage And Hybrid Backlogging Function.

ABSTRACT:

Perishable inventory system become highly Complex when economic fluctuations, storage limitations, and Customer shortage behavior act simultaneously. This research presents an integrated inventory model for perishable goods under the combined effects of inflation, limited storage capacity, and a hybrid backlogging strategy. Inflation is incorporated through a continuous discounting approach to reflect the changing value of money over time. The proposed backlogging mechanism depends on both waiting time and selling price, representing practical customer behavior during shortages. A portion of unmet demand is backordered based on delay tolerance and price sensitivity, while the remaining demand is considered lost sales. The total cost function includes ordering, holding, deterioration, shortage, and inflation-adjusted purchasing costs. The objective is to determine the optimal replenishment cycle, order quantity, and shortage duration that minimize overall system cost. Numerical examples and sensitivity analysis highlight the significant impact of inflation, warehouse limitations, and hybrid backlogging on optimal inventory decisions.

Keyword: Perishable inventory, Limited Storage Capacity, Hybrid backlogging, Time-price dependent shortage, Supply chain management

Introduction

Perishable inventory management has attracted substantial attention due to its relevance in food, pharmaceutical, and cold-chain sectors where deterioration significantly affects inventory decisions. The concept of deterioration was first introduced by Ghare and Schrader [4], and later extended by Covert and Philip [2], forming the theoretical foundation of perishable inventory models. With changing economic conditions, inflation was incorporated into inventory analysis; Wee [8] developed a lot-sizing model using a discounted cash flow approach, while Hwang and Shinn [5] studied joint pricing and replenishment policies under inflationary environments. However, these studies largely overlooked warehouse capacity constraints and realistic shortage behavior. Later, Wu et al. [17] and Singh and Sharma [13] highlighted the influence of waiting time and price sensitivity on customer backlogging decisions Nevertheless, a comprehensive framework integratingdeterioration, inflation, limited storage, and hybrid backlogging remains limited, motivating the present research

Objective. This study proposes an integrated perishable inventory model under inflation and capacity constraints to determine optimal replenishment and shortage policies that minimize total system cost through hybrid backlogging**Review of Literature.** The early development of deteriorating inventory models began with Ghare and Schrader (1963)[4] who incorporate a constant deterioration rate into the EOQ frame work. Covert and Philip (1973)[2] further generalized the deterioration process by introducing variable decay patterns. These models established the theoretical basis for perishable inventory research but ignored economic inflation and storage limitations. The role of inflation in inventory management was explored by Wee (1993)[8] and Hwang and Shinn (1997)[5], who demonstrated that inflation significantly affects ordering and holding costs. Later, Jaggi and Khanna (2009)[6] and Teng et al. (2011)[14] Combined deterioration with inflationary environments to provide more realistic Cost Structures. However, these studies generally assumed unlimited storage Capacity. Shortage and back logging policies were analyzed by Nahmis (1982)[10] and Ravid (1987)[12], primarily focusing on waiting-time dependent partial backlogging. More recent contributions by Wu et al. (2014)[17] and Singh and sharma (2020)[13] introduced time-dependent and price-Sensitive backlogging functions, highlighting that customer willingness to wait decreases with increased shortage duration or higher prices. Nevertheless these models often considered time-based or price-based backlogging Separately rather than in anintegrated hybrid form. Storage Capacity Constraints were incorporated by Mandal and Pal (1998)[9] and khanna et al. (2011)[7], who developed models under limited warehouse Space Taleizadeh et al. (2016) further examined shortage behavior under storage restrictions: However, the simultaneous integration of deterioration, inflation -adjusted Cost Structures, limited storage Capacity, and hybrid backlogging has not been comprehensively addressed in a single optimization Framework.Therefore, the present study attempts to bridge this gap by proposing an integrated and realistic perishable inventory model that captures there interacting factors.

Assumption & Notations

Notation

1. T - Length of one replenishment Cycle
2. t_1 - Inventory depletion time (no shortage period ends) 3. $\tau =$ T- t_1 Shortage duration
4. D(t) = Time-dependent demand rate
5. D_0 = Time Initial demand rate at time O
6. $\gamma =$ Demand growth parameter
7. $\theta =$Deterioration rate of perishable item
8. I(t) = Inventory level at time t
9. $p(T,P) =$Hybridbacklogging fraction depending on time & price
10. p_0 = Minimum backlogging proportion.
11. $\alpha =$ Time Sensitivity of Customers
12. $\beta =$ price- Sensitivity factor
13. S_{max} = Maximum Storage Capacity
14. C_h = Holding cost per unit
15. C_b = Backlogging cost per unit
16. C_l = Lost-Sales Penalty
17. C_p = Purchasing Cost per unit
18. K = ordering/ setup cost
19. $\gamma =$ Continuous inflation/discount rate
20. TC(T) = Total cost per cycle

Assumptions

1. Demand Varies with time and increases linearly

$$D(t) = D_0(1+\gamma t)$$

2. Inventory items deteriorate at a constant rate θ
3. Inflation is incorporated using Continuous discounting:

$$e^{-\gamma t}$$

4. Storage space is limited, hence maximum permissible inventory cannot sexceedS_{max}
5. Shortage is allowed and unmet demand is partial backlogged
6. Backlogging proportion depends simultaneously on waiting time and selling price:

$$B(t) = \frac{1}{1+\beta(T_s - t)} + \alpha e^{-mt}$$

7. The remaining fraction (1-p) is considered lost sales.
8. Lead time is negligible and replenishment is instantaneous.
9. All costs are inflation-adjusted and measure in present Value terms

Analytical Expression

Context I:- During the inventory accumulation stage, $[0,t_1]$ the inventory reduction occur due to demand fulfillment and deterioration, Consequently, it is modeled by the differential Equation ,

$$\frac{dI(t)}{dt} = -D(t) - \theta I(t), \quad 0 \le t \le t_1 \qquad (1)$$

this is eqn ------------------(1)

subject to boundary condition I(0)= I_0

The solution of eqn (1) is

$$I(t) = (I_0 + \frac{D_0}{\theta} - \frac{D_0\gamma}{\theta^2})e^{-\theta t} - \frac{D_0}{\theta}t + D_0\left(\frac{\gamma}{\theta^2} - \frac{1}{\theta}\right) \qquad (2)$$

Context II:- Inventory state with backorder During the backorder stage $[t_1, T]$ the demand is partial deferred and the inventory level is adequately maintained,

$$\frac{dI(t)}{dt} = \frac{1}{1+\beta(T_s - t)} + \alpha e^{-mt} \qquad ----(3)$$

This is eqn (3).

Subject to the boundary condition T(t)=0,t=T

The solution of eqn (3) is,

$$I(t)=\frac{b}{\delta}(T-t)+\left(\frac{a}{\delta}-\frac{b}{\delta^2}\right)[\log(1+\delta t)-\log(1+\delta t)-----(4)$$

Consequently, the overall cost per replenishment cycle is composed of the following parts.

1. Holding cost per cycle;

$$H_c=h\int_0^{t_1}I(t)e^{-rt}\,dt$$

$$= \text{h}\left[\frac{A}{\gamma-\theta}\left(e^{(\gamma-\theta)t_1}\right)+\frac{\beta}{\gamma}\left(e^{\gamma t_1}\right)\right]$$

$$=\text{h}[\frac{I_0+D_0-\frac{D_0\gamma}{\theta_2}}{\gamma-\theta}\left(e^{(\gamma-\theta)t_1}-1\right)+\frac{\frac{-D_0\gamma}{\theta}+\frac{D_0}{\theta}\left(\frac{\gamma}{\theta_2}-1\right)}{\gamma}\left(e^{-rt_1}\right)]$$

2. Deterioration cost per cycle;

$$D_c=\ C_d\theta\int_0^{t_1}I(t)e^{-rt}dt$$

$$=C_d[\frac{\theta\left(I_0+\frac{D_0}{\theta}-\frac{D_0}{\theta^2}\right)}{\gamma-\theta}\left(e^{(\gamma-\theta)t_1}-1\right)-\frac{D_0}{\gamma}\left(e^{\gamma t_1}(\gamma t_1-1)+1\right)+\frac{D_0(\gamma-\theta)}{\theta\gamma}\left(e^{\gamma t_1}-1\right)$$

3. Backlogging cost per cycle;

$$B_c=\ C_b\int_{t_1}^{T}B(t)e^{-r(t-\frac{T}{2})}dt$$

$$=C_b(\beta_0+\beta_1 p)e^{\frac{\gamma t}{\alpha}}.\frac{e^{\frac{\gamma t}{\alpha}}}{\alpha}[\left(\frac{-\gamma}{\alpha}-\gamma T\right)-\left(\frac{-\gamma}{\alpha}-\gamma t_1\right)]$$

4. Lost sale cost per cycle;

$$L_c=\ C_l\int_{t_1}^{T}(1-B(t)D(t))e^{-\gamma(t^1+\frac{T}{2})}\ dt$$

$$=C_l[\beta(T\left(T-t_1+\frac{\gamma T}{2}(T^2-t_1^2)-\frac{1}{2}(T^2-t_1^2)-\frac{\gamma}{3}(T^2-t_1^2)\right)-\alpha(\frac{e^{-mt_1-e^{-mt}}}{m}+\qquad\gamma(e^{-mT}\left(\frac{T}{m}+\frac{1}{m^2}\right)+$$
$$e^{-mt_1}\left(\frac{t_1}{m}+\frac{1}{m^2}\right)]$$

5. Purchasing cost per cycle;

$$P_c=\ C_p\int_0^{T}D(t)e^{-\gamma t}\ dt$$

$$=\ C_pD_0[\frac{1}{r}+\frac{\gamma}{r^2}+e^{-rt}\left(\frac{1}{r}+\frac{\gamma t}{r}+\frac{\gamma}{r^2}\right)]_0^{\mathrm{T}}$$

$$=\ C_pD_0(T-\frac{\gamma T^2}{2})$$

6. Ordering cost per cycle;

$$O_c=\ C_0$$

7. Shortage cost per cycle;

$$S_c=\ C_s\int_{t_1}^{T}(-I(t)dt$$

$$=C_sI_0+\frac{D_0}{\theta}-\frac{D_0\gamma}{\theta^2}\left(e^{-\theta T}-e^{-\theta b_1}\right)-\ C_s\left(\frac{-D_0\gamma}{2\theta}\right)(T^2-b_1^2)-C_sD_0\left(\frac{\gamma}{\theta^2}-\frac{1}{\theta}\right)(T-b_1)$$

Consequently the total cost per cycle is given;

$$T_{cycle}\ =\ H_{cycle}\ +D_{cycle}\ +B_{cycle}\ +\ L_{cycle}\ +P_{cycle}\ +O_{cycle}\ +S_{cycle}$$

$$=\text{h}\left[\frac{I_0+D_0-\frac{D_0\gamma}{\theta_2}}{\gamma-\theta}\left(e^{(\gamma-\theta)t_1}-1\right)+\qquad\frac{\frac{-D_0\gamma}{\theta}+\frac{D_0}{\theta}\left(\frac{\gamma}{\theta_2}-1\right)}{\gamma}\left(e^{-rt_1}\right)\right]$$
$$+C_d[\frac{\theta\left(I_0+\frac{D_0}{\theta}-\frac{D_0}{\theta^2}\right)}{\gamma-\theta}\left(e^{(\gamma-\theta)t_1}-1\right)-\frac{D_0}{\gamma}\left(e^{\gamma t_1}(\gamma t_1-1)+1\right)+\frac{D_0(\gamma-\theta)}{\theta\gamma}\left(e^{\gamma t_1}-1\right)+C_b(\beta_0+\ \beta_1 p)e^{\frac{\gamma t}{\alpha}}.\frac{e^{\frac{\gamma t}{\alpha}}}{\alpha}[\left(\frac{-\gamma}{\alpha}-\gamma T\right)-$$
$$\left(\frac{-\gamma}{\alpha}-\gamma t_1\right)]+C_l[\beta(T\left(T-t_1+\frac{\gamma T}{2}(T^2-t_1^2)-\frac{1}{2}(T^2-t_1^2)-\frac{\gamma}{3}(T^2-t_1^2)\right)-\ \alpha(\frac{e^{-mt_1-e^{-mt}}}{m}+\gamma(e^{-mT}\left(\frac{T}{m}+\frac{1}{m^2}\right)+$$
$$e^{-mt_1}\left(\frac{t_1}{m}+\frac{1}{m^2}\right)]+\ C_pD_0(T-\frac{\gamma T^2}{2})+C_sI_0+\frac{D_0}{\theta}-\frac{D_0\gamma}{\theta^2}\left(e^{-\theta T}-e^{-\theta b_1}\right)-C_s\left(\frac{-D_0\gamma}{2\theta}\right)(T^2-b_1^2)-C_sD_0(\frac{\gamma}{\theta^2}-\frac{1}{\theta})(T-b_1)$$

Sample calculation:- Consider an inventory system characterized by data provided in units, then a = 60, b = 0.8, $\theta = 0.025, \delta = 0.12, S = 900, O_c = 250, C_p = 12, C_h = 0.6, C_s = 6, Base\ TC = 1305$

***Outcome of adjustments in the rate of deterioration $\boldsymbol{\theta}$**

Index - I

$\boldsymbol{\theta}$	$\boldsymbol{T_0}$	$\boldsymbol{T_1}$	$\boldsymbol{T}$	$\boldsymbol{TC}$
0.012	7.95	11.55	19.50	1125.40
0.025	8.30	12.05	20.35	1305.50
0.060	8.68	12.70	21.38	1402.75
0.110	9.10	13.40	22.50	1528.60

Deteriorating rate ($\boldsymbol{\theta}$)

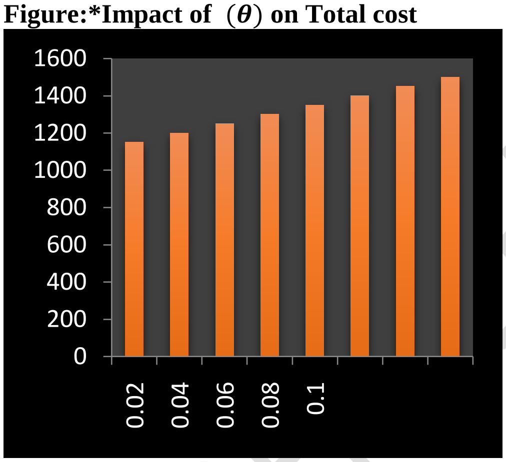
***Outcome of adjustments in the parameter of backlogging $\boldsymbol{\delta}$**

Index – II

$\boldsymbol{\delta}$	$\boldsymbol{T_0}$	$\boldsymbol{T_1}$	$\boldsymbol{T}$	$\boldsymbol{TC}$
0.04	8.05	11.90	19.95	1188.20
0.12	8.30	12.50	20.45	1305.50
0.22	8.60	12.70	21.45	1360.95
0.33	8.95	13.25	22.20	1490.30

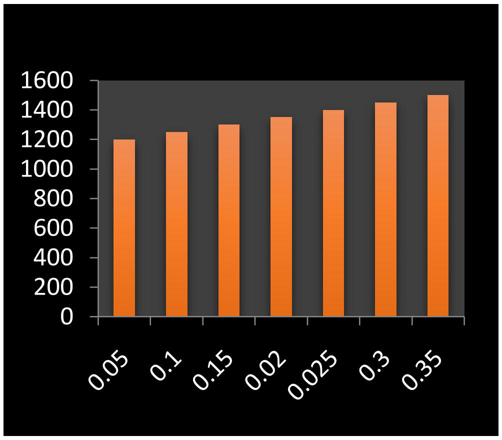
Backlogging Parameter ($\boldsymbol{\delta}$)

Figure:*Impact of ($\boldsymbol{\theta}$) on Total cost



Deterioration rate ($\boldsymbol{\theta}$)

Figure(b):*Impact of ($\boldsymbol{\delta}$) on Total cost



Backlogging Parameter (δ)

Parametric Analysis:

Beginning at Index I and Index II, we see the parametric to both the deterioration rate θ and the backlogging parameter δ. An increase in θ intensifies inventory loss, when consequently raise the total cost and replenishment decisions. In contrast δ moderately influence system performance, higher Value duration and gradually increases the cost. Overall the parameter δ is more responsive than the parameter θ

Conclusion:

This study presents an integrated perishable inventory model incorporating inflation, limited storage, deterioration, and hybrid backlogging. Results highlight their significant impact on optimal replenishment cycles, order quantities, and shortage periods. By capturing realistic customer behavior, the model serves as a robust decision-making tool for efficient inventory planning and cost control under uncertain economic conditions.

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