

REVIEWER'S REPORT

Manuscript No.: JNCS-025

Title: A Second Derivative Hybrid Block Method with Variational Stability Parameters for the Direct Solution of Second-Order Initial Value Problems in Ordinary Differential Equations

Recommendation:

Accept as it is

Accept after minor revision.....

Accept after major revision✓.....

Do not accept (*Reasons below*)

Rating	Excel.	Good	Fair	Poor
Originality		✓		
Techn. Quality		✓		
Clarity		✓		
Significance		✓		

Reviewer's ID: JPR-098

Detailed Reviewer's Report

Recommendation: Accept after Major Revision

Summary of the paper

The manuscript proposes a second-derivative hybrid block method with variable stability parameters for the direct numerical solution of second-order initial value problems in ordinary differential equations. The method is derived using interpolation and collocation, and its stability properties are analyzed. Numerical examples are provided to demonstrate accuracy and efficiency.

Overall assessment

The topic is relevant to numerical analysis and ODE solvers, and the idea of incorporating variable stability parameters into a hybrid block method is interesting. However, the manuscript currently suffers from major issues in mathematical presentation, notation consistency, derivation clarity, and experimental validation. The paper requires substantial revision before it can be considered for publication.

REVIEWER'S REPORT**Major revision points****1. Notation and mathematical consistency**

The manuscript contains numerous inconsistencies in notation, indexing, and equation formatting. For example, equations (2.3)–(2.5) mix indices and superscripts unclearly, and several expressions appear malformed. The mathematical presentation must be carefully revised for correctness and readability.

2. Clarity of derivation

The derivation of the block method is extremely difficult to follow due to long, unstructured algebraic expansions. Key steps should be explained conceptually, intermediate matrices should be simplified, and unnecessary symbolic expansions should be moved to an appendix.

3. Definition of stability parameters

The paper repeatedly refers to “variational stability parameters,” but their meaning, motivation, and practical role are not clearly defined. The authors should explicitly define the parameters, explain how they are chosen, and discuss their effect on stability and accuracy.

4. Stability analysis needs strengthening

The claims of $A(\alpha)$ -stability and A -stability are not rigorously justified. The manuscript should include a clear derivation of the stability function, the test equation used, and the precise conditions under which the method is stable.

5. Convergence analysis is incomplete

The convergence proof is not sufficiently detailed. The authors should present consistency, zero-stability, and convergence arguments in a structured manner, preferably using standard theorems from multistep method analysis.

REVIEWER'S REPORT**6. Numerical experiments are insufficient**

Only a limited number of test problems are presented, and the comparison is mainly visual (“overlap with the exact solution”). The paper should include quantitative error tables, convergence orders, CPU time comparisons, and comparisons with established methods.

7. Reproducibility and implementation details

Important implementation details are missing, including the algorithmic procedure, initialization strategy, step-size selection, and how the nonlinear systems are solved in practice. Pseudocode or a concise algorithm section would improve reproducibility.

8. Literature review and positioning

The introduction cites many works but does not clearly position the proposed method relative to existing second-derivative block methods. The novelty and advantages over previous approaches should be stated more explicitly.

Minor revision points

1. Improve grammar, punctuation, and sentence structure throughout the manuscript.
2. Standardize notation for grid points, derivatives, and coefficients.
3. Ensure all equations are properly numbered and referenced in the text.
4. Correct formatting issues in matrices and long equations.
5. Expand figure captions and explain what the figures demonstrate.
6. Define all symbols immediately after first use.
7. Remove repetitive statements in the introduction.
8. Provide units or scaling information for numerical examples where relevant.
9. Ensure references are consistently formatted and complete.
10. Clarify whether the method is self-starting or requires external starting values.

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REVIEWER'S REPORT

Recommendation

Major Revision

The manuscript has potential, but substantial improvements are required in mathematical rigor, presentation clarity, and numerical validation before it can be considered for publication.